

## Exercise 154

For the following exercises, verify that each equation is an identity.

$$\frac{\tan \theta - \cot \theta}{\sin \theta \cos \theta} = \sec^2 \theta - \csc^2 \theta$$

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### Solution

$$\begin{aligned}\frac{\tan \theta - \cot \theta}{\sin \theta \cos \theta} &\stackrel{?}{=} \sec^2 \theta - \csc^2 \theta \\ \frac{\tan \theta}{\sin \theta \cos \theta} - \frac{\cot \theta}{\sin \theta \cos \theta} &\stackrel{?}{=} \sec^2 \theta - \csc^2 \theta \\ \frac{1}{\sin \theta \cos \theta} \left( \frac{\sin \theta}{\cos \theta} \right) - \frac{1}{\sin \theta \cos \theta} \left( \frac{\cos \theta}{\sin \theta} \right) &\stackrel{?}{=} \sec^2 \theta - \csc^2 \theta \\ \frac{1}{\cos^2 \theta} - \frac{1}{\sin^2 \theta} &\stackrel{?}{=} \sec^2 \theta - \csc^2 \theta \\ \sec^2 \theta - \csc^2 \theta &= \sec^2 \theta - \csc^2 \theta\end{aligned}$$

This is a true statement, so the identity is verified.